

Roll No \_\_\_\_\_ (To be filled in by the candidate)

(Academic Sessions 2017 – 2019 to 2019 – 2021)

MATHEMATICS

221-(INTER PART – II)

Time Allowed : 30 Minutes

Q.PAPER – II ( Objective Type )

GROUP – I

Maximum Marks : 20

PAPER CODE = 8195

Note : Four possible answers A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer-book. Cutting or filling two or more circles will result in zero mark in that question.

1-1	$\int (2x+3)^{\frac{1}{2}} dx = :$ <p>(A) <math>\frac{(2x+3)^{\frac{3}{2}}}{2} + c</math>                      (B) <math>\frac{1}{3}(2x+3)^{\frac{3}{2}} + c</math></p> <p>(C) <math>\frac{1}{2}(2x+3)^{\frac{1}{3}} + c</math>                      (D) <math>\frac{1}{3}(2x+3)^{\frac{-1}{2}} + c</math></p>
2	Distance between A ( 3 , 1 ) and B ( - 2 , - 4 ) is : (A) $\sqrt{17}$ (B) $5\sqrt{2}$ (C) $\sqrt{26}$ (D) $2\sqrt{5}$
3	If $f(x) = \frac{x}{x^2 - 4}$ then range of $f(x)$ is : (A) All real number                      (B) Rational number (C) All negative real number                      (D) Integer
4	Slope 'm' through A(x <sub>1</sub> , y <sub>1</sub> ) B(x <sub>2</sub> , y <sub>2</sub> ) is : (A) $\frac{x_2 - x_1}{y_2 - y_1}$ (B) $\frac{x_2 + x_1}{y_2 - y_1}$ (C) $\frac{y_2 - y_1}{x_2 - x_1}$ (D) $\frac{y_1 - y_2}{x_1 + x_2}$
5	$\lim_{x \rightarrow 0} \frac{\sin ax}{\sin bx} = :$ <p>(A) <math>\frac{b}{a}</math>                      (B) a                      (C) <math>\frac{a}{b}</math>                      (D) <math>\frac{1}{b}</math></p>
6	$\int (a-2x)^{\frac{3}{2}} dx = :$ <p>(A) <math>\frac{1}{5}(a-2x)^{\frac{3}{2}} + c</math>                      (B) <math>\frac{1}{5}(a-2x)^{\frac{5}{2}} + c</math></p> <p>(C) <math>-\frac{1}{5}(a-2x)^{\frac{5}{2}} + c</math>                      (D) <math>-\frac{3}{5}(a-2x)^{\frac{5}{2}} + c</math></p>
7	$\int \sec x dx = :$ <p>(A) <math>\sec x + \tan x</math>                      (B) <math>\sec^2 x</math></p> <p>(C) <math>\ln \sec x - \tan x </math>                      (D) <math>\ln \sec x + \tan x  + c</math></p>
8	If $f(x) = \frac{1}{x^m}$ then $f'(x) = :$ (A) $-xm^{-1}$ (B) $-mx^{-m-1}$ (C) $-mx^{-m+1}$ (D) $-m^{-1}x$

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(2)

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1-9	Midpoint of the line segment joining A (3, 1) and B (-2, -4) is : (A) $\left(\frac{1}{2}, -\frac{3}{2}\right)$ (B) $\left(\frac{5}{2}, \frac{5}{2}\right)$ (C) $\left(\frac{1}{2}, \frac{3}{2}\right)$ (D) $\left(\frac{1}{2}, \frac{5}{2}\right)$
10	The derivative of $\frac{1}{1+x}$ is : (A) $x$ (B) $1+x$ (C) $(1+x)^{-2}$ (D) $-1(1+x)^{-2}$
11	In circle $x^2 + y^2 + 2gx + 2fy + c = 0$ , the radius is : (A) $\sqrt{g^2 + f^2 + c}$ (B) $g^2 + f^2 - c$ (C) $\sqrt{g^2 + f^2 - c}$ (D) $g^2 + f^2 + c$
12	$x = 5$ is the solution of inequality : (A) $2x - 3 > 0$ (B) $2x + 3 < 0$ (C) $x + 4 < 0$ (D) $x + 3 < 0$
13	In vectors $\vec{a} \times \vec{b} = :$ (A) $\vec{b} \times \vec{a}$ (B) $-\vec{b} \times \vec{a}$ (C) $-\vec{b}$ (D) $-\vec{a} \times \vec{b}$
14	In equation of circle $x^2 + y^2 = r^2$ the centre of circle is : (A) $(x, y)$ (B) $(0, 0)$ (C) $(1, 0)$ (D) $(0, r)$
15	Magnitude of vector $\vec{u} = 2i - 7j$ is : (A) $\sqrt{53}$ (B) $\sqrt{55}$ (C) $\sqrt{48}$ (D) $\sqrt{52}$
16	$\frac{d}{dx} (\cos^{-1} x) = :$ (A) $\frac{1}{\sqrt{1-x^2}}$ (B) $\frac{-1}{\sqrt{1-x^2}}$ (C) $\frac{1}{\sqrt{1+x^2}}$ (D) $\frac{1}{1+x^2}$
17	$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$ is Maclaurin series for : (A) $e^x$ (B) $\sqrt{1+x}$ (C) $\cos x$ (D) $\sin x$
18	The vector $\vec{PQ}$ through P (0, 5) and Q (-1, -6) is : (A) $[-1, 11]$ (B) $[-1, -11]$ (C) $[0, 11]$ (D) $[1, 1]$
19	$\frac{d}{dx} \tan^{-1} x = :$ (A) $\frac{1}{1-x^2}$ (B) $\frac{1}{\sqrt{1-x^2}}$ (C) $\frac{1}{\sqrt{1+x^2}}$ (D) $\frac{1}{1+x^2}$
20	The focus of parabola $y^2 = 4ax$ is : (A) $(0, a)$ (B) $(-a, 0)$ (C) $(a, 0)$ (D) $(0, -a)$

## SECTION – I

## 2. Write short answers to any EIGHT (8) questions :

16

- (i) Find the domain and range of the function  $g$  defined by :  $g(x) = \sqrt{x^2 - 4}$
- (ii) The real valued functions  $f$  and  $g$  are given. Find  $f \circ g(x)$ , if  
 $f(x) = 3x^4 - 2x^2$  and  $g(x) = \frac{2}{\sqrt{x}}$ ,  $x \neq 0$
- (iii) Evaluate  $\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\theta}$
- (iv) Evaluate  $\lim_{x \rightarrow 1} \frac{x^3 - 3x^2 + 2x - 1}{x^3 - x}$
- (v) Find  $\frac{dy}{dx}$  if  $x^2 - 4xy - 5y = 0$
- (vi) Differentiate w.r.t. 'x'  $\cot^{-1}\left(\frac{x}{a}\right)$
- (vii) Find  $f'(x)$  if  $f(x) = \sqrt{\ln(e^{2x} + e^{-2x})}$
- (viii) Find  $y_2$  if  $x^3 - y^3 = a^3$
- (ix) Prove that  $\frac{d}{dx} (\operatorname{cosec}^{-1} x) = \frac{-1}{|x| \sqrt{x^2 - 1}}$
- (x) Differentiate  $\frac{2x - 1}{\sqrt{x^2 + 1}}$
- (xi) Find the interval in which function is increasing or decreasing :  
 $f(x) = \cos x$   $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
- (xii) Find  $y_4$  if  $y = \sin 3x$

## 3. Write short answers to any EIGHT (8) questions :

16

- (i) Use differentials to approximate the value of  $\sqrt[4]{17}$
- (ii) Solve  $\int \frac{dx}{\sqrt{x+1} - \sqrt{x}}$
- (iii) Evaluate  $\int \frac{\cot \sqrt{x}}{\sqrt{x}} dx$
- (iv) Solve  $\int \frac{\sec^2 x}{\sqrt{\tan x}} dx$
- (v) Solve  $\int e^{2x} [-\sin x + 2 \cos x] dx$
- (vi) Evaluate  $\int_0^{\frac{\pi}{4}} \sec x (\sec x + \tan x) dx$
- (vii) Solve the differential equation  $\frac{1}{x} \frac{dy}{dx} = \frac{1}{2}(1 + y^2)$
- (viii) Evaluate  $\int x \ln x dx$
- (ix) The points  $A(-5, -2)$ ,  $B(5, -4)$  are ends of a diameter of a circle. Find centre and radius of it.

(Turn Over)

3. (x) Transform the equation  $5x - 12y + 39 = 0$  into normal form.  
 (xi) Find  $k$  so that the lines joining  $A(7, 3)$ ,  $B(k, -6)$  and  $C(-4, 5)$ ,  $D(-6, 4)$  are parallel.  
 (xii) Find the lines represented by  $2x^2 + 3xy - 5y^2 = 0$

4. Write short answers to any NINE (9) questions :

18

- (i) Graph the inequality  $5x - 4y \leq 20$   
 (ii) Find the equation of the circle with ends of diameter at  $(-3, 2)$  and  $(5, -6)$   
 (iii) Find the centre of the circle  $4x^2 + 4y^2 - 8x + 12y - 25 = 0$   
 (iv) Find the length of the tangent from the point  $(-5, 10)$  to the circle  $5x^2 + 5y^2 + 14x - 12y - 10 = 0$   
 (v) Find the coordinates of the points of intersection of the line  $x + 2y = 6$  with the circle  $x^2 + y^2 - 2x - 2y - 39 = 0$   
 (vi) Find the vertex of the parabola  $x^2 = 4(y - 1)$   
 (vii) Find the foci of the hyperbola  $\frac{y^2}{16} - \frac{x^2}{9} = 1$   
 (viii) Find a unit vector in the direction of  $\underline{v} = -\frac{\sqrt{3}}{2}\underline{i} - \frac{1}{2}\underline{j}$   
 (ix) Find a vector whose magnitude is 4 and is parallel to  $2\underline{i} - 3\underline{j} + 6\underline{k}$   
 (x) If  $\underline{v}$  is a vector for which  $\underline{v} \cdot \underline{i} = 0$ ,  $\underline{v} \cdot \underline{j} = 0$  and  $\underline{v} \cdot \underline{k} = 0$ , find  $\underline{v}$   
 (xi) If  $\underline{a} + \underline{b} + \underline{c} = 0$ , then prove that  $\underline{a} \times \underline{b} = \underline{b} \times \underline{c} = \underline{c} \times \underline{a}$   
 (xii) Find the volume of parallelepiped for which the vectors  $\underline{u} = \underline{i} - 4\underline{j} - \underline{k}$ ,  $\underline{v} = \underline{i} - \underline{j} - 2\underline{k}$  and  $\underline{w} = 2\underline{i} - 3\underline{j} + \underline{k}$  are three edges.  
 (xiii) Give a force  $\underline{F} = 2\underline{i} + \underline{j} - 3\underline{k}$  acting at a point  $A(1, -2, 1)$ . Find the moment of  $\underline{F}$  about the point  $B(2, 0, -2)$

## SECTION - II

Note : Attempt any THREE questions.

5. (a) Discuss the continuity of  $f(x)$  at  $x = c$   $f(x) = \begin{cases} 3x - 1 & \text{if } x < 1 \\ 4 & \text{if } x = 1 \\ 2x & \text{if } x > 1 \end{cases}$ ,  $c = 1$  5  
 (b) Show that  $\frac{dy}{dx} = \frac{y}{x}$  if  $\frac{y}{x} = \tan^{-1} \frac{x}{y}$  5  
 6. (a) Evaluate  $\int x \sin^{-1} x \, dx$  5  
 (b) Find the interior angles of the triangle with vertices  $A(6, 1)$ ,  $B(2, 7)$ ,  $C(-6, -7)$  5  
 7. (a) Evaluate  $\int_0^{\frac{\pi}{4}} \frac{1}{1 + \sin x} \, dx$  5  
 (b) Minimize  $z = 2x + y$  subject to constraints  $x + y \geq 3$ ,  $7x + 5y \leq 35$ ;  $x \geq 0$ ,  $y \geq 0$  5  
 8. (a) Prove that in any triangle ABC  $b^2 = c^2 + a^2 - 2ca \cos B$ . 5  
 (b) Find the length of the chord cut off from the line  $2x + 3y = 13$  by the circle  $x^2 + y^2 = 26$  5  
 9. (a) If  $y = (\cos^{-1} x)^2$  then prove that  $(1 - x^2)y_2 - xy_1 - 2 = 0$  5  
 (b) Find the points of intersection of the given conic  $\frac{x^2}{18} + \frac{y^2}{8} = 1$  and  $\frac{x^2}{3} - \frac{y^2}{3} = 1$  5

(Academic Sessions 2017 – 2019 to 2019 – 2021)

**MATHEMATICS**

221-(INTER PART – II)

Time Allowed : 30 Minutes

Q.PAPER – II ( Objective Type )

GROUP – II

Maximum Marks : 20

**PAPER CODE = 8198**

Note : Four possible answers A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer-book. Cutting or filling two or more circles will result in zero mark in that question.

1-1	The derivative of $\frac{1}{1+x}$ is :	(A) $x$	(B) $1+x$	(C) $(1+x)^{-2}$	(D) $-1(1+x)^{-2}$
2	$\int \cos x \, dx = :$	(A) $1 - \sin^2 x$	(B) $\sqrt{1 - \sin^2 x}$	(C) $\sin x$	(D) $-\sin x$
3	$\int_1^2 (x^2 + 1) \, dx = :$	(A) $\frac{10}{3}$	(B) $\frac{3}{10}$	(C) $\pi$	(D) $\frac{\pi}{2}$
4	If $y = \cot^{-1} x$ , then $\frac{dy}{dx} = :$	(A) $\frac{1}{1-x^2}$	(B) $\frac{-1}{1+x^2}$	(C) $\frac{1}{x^2-1}$	(D) $\frac{1}{x^2+1}$
5	The derivative of $\ln(\tanh x)$ is :	(A) $\frac{1}{\tanh x}$	(B) $\frac{\sec^2 x}{\tanh x}$	(C) $\sec^2 x$	(D) $\sec x$
6	$x = at^2$ and $y = 2at$ are parametric equations of :	(A) Parabola	(B) Ellipse	(C) Circle	(D) Hyperbola
7	If $y^2 + x^2 = a^2$ , then $\frac{dy}{dx} = :$	(A) $-\frac{x}{y}$	(B) $-\frac{y}{x}$	(C) $\frac{x}{y}$	(D) $\frac{y}{x}$
8	The order of $\frac{dy}{dx} = \frac{4}{3}x^3 + x - 3$ is :	(A) 1	(B) $\frac{3}{4}$	(C) $\frac{4}{3}$	(D) -3
9	$\int_a^x 3x^2 \, dx = :$	(A) $x^3 + a^3$	(B) $x^3 - a^3$	(C) $3x^3$	(D) $x^3$

( Turn Over )

1-10	If $\theta$ is measured in radian then $\lim_{\theta \rightarrow 0} \frac{\sin 7\theta}{\theta} = :$ (A) 7 (B) $\frac{1}{7}$ (C) $\frac{7\pi}{22}$ (D) $\frac{7\pi}{12}$
11	The measure of the angle between the lines $ax^2 + 2hxy + by^2 = 0$ is given by $\tan \theta = :$ (A) $\frac{\sqrt{h^2 - ab}}{a - b}$ (B) $\frac{2\sqrt{h^2 - ab}}{a + b}$ (C) $\frac{h^2 - ab}{a + b}$ (D) $\infty$
12	If $\vec{a} = \hat{i} - \hat{j}$ and $\vec{b} = \hat{j} + \hat{k}$ then $\vec{a} \cdot \vec{b} = :$ (A) 0 (B) 1 (C) -1 (D) $\sqrt{2}$
13	The feasible solution which maximize or minimize the objective function is called : (A) Boundary (B) Half plane (C) Optimal solution (D) Initial values
14	The value of c for $\frac{y^2}{16} - \frac{x^2}{49} = 1$ is : (A) 16 (B) 49 (C) 65 (D) $\sqrt{65}$
15	The equation of a straight line represented by $x \cos \alpha + y \sin \alpha = P$ is called : (A) Normal form (B) Angular form (C) Symmetric form (D) P - form
16	The unit vector in the direction of $\vec{v} = [3, -4]$ : (A) $5[3, -4]$ (B) $\frac{1}{5}[3, -4]$ (C) $\hat{i}$ (D) $\hat{j}$
17	The points A $(-5, -2)$ , B $(5, -4)$ are ends point of a diameter of the circle. The centre will be : (A) $(0, 3)$ (B) $(0, -3)$ (C) $(5, 2)$ (D) $(-5, 4)$
18	$xy = 0$ represents : (A) A pair of lines (B) Hyperbola (C) Parabola (D) Ellipse
19	The projection of $\vec{v}$ along $\vec{u}$ is : (A) $\frac{\vec{u} \cdot \vec{v}}{ u }$ (B) $\frac{\vec{u} \cdot \vec{v}}{ v }$ (C) $\frac{\vec{u} \cdot \vec{v}}{ u  v }$ (D) $\frac{\vec{u} \cdot \vec{v}}{ u  +  v }$
20	An angle inscribed in a semi-circle is : (A) 0 (B) $\frac{\pi}{2}$ (C) $\pi$ (D) $2\pi$

Roll No \_\_\_\_\_ (To be filled in by the candidate)

(Academic Sessions 2017 – 2019 to 2019 – 2021)

MATHEMATICS 221-(INTER PART – II)

PAPER – II (Essay Type)

GROUP – II

Time Allowed : 2.30 hours

Maximum Marks : 80

## SECTION – I

## 2. Write short answers to any EIGHT (8) questions :

16

- (i) Express the area  $A$  of a circle as a function of its circumference  $C$ .
- (ii) For the real-valued function  $f(x) = \frac{2x+1}{2x-1}$ ,  $x > 1$ . Find  $f^{-1}(x)$
- (iii) Evaluate  $\lim_{x \rightarrow 3} \frac{x-3}{\sqrt{x}-\sqrt{3}}$
- (iv) Find the domain and range of  $g(x) = |x-3|$
- (v) If  $y = \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2$ , find  $\frac{dy}{dx}$
- (vi) Find  $\frac{dy}{dx}$  if  $xy + y^2 = 2$
- (vii) Differentiate  $\sin x$  w.r.t.  $\cot x$
- (viii) Find  $\frac{dy}{dx}$  if  $y = x^2 \ln \frac{1}{x}$
- (ix) Find  $y_2$  if  $y = x^2 \cdot e^{-x}$
- (x) If  $y = \ln(\tanh x)$ , find  $\frac{dy}{dx}$
- (xi) Find  $\frac{dy}{dx}$  if  $y = (x^2 + 5)(x^3 + 7)$
- (xii) Find  $f'(x)$  if  $f(x) = \sqrt{\ln(e^{2x} + e^{-2x})}$

## 3. Write short answers to any EIGHT (8) questions :

16

- (i) Use differential to find  $\frac{dy}{dx}$  for  $xy + x = 4$
- (ii) Evaluate the integral  $\int \frac{3x+2}{\sqrt{x}} dx$
- (iii) Evaluate  $\int \frac{x+b}{(x^2+2bx+c)^{1/2}} dx$
- (iv) Evaluate  $\int e^x (\cos x + \sin x) dx$
- (v) Evaluate  $\int \frac{(a-b)x}{(x-a)(x-b)} dx$
- (vi) Evaluate  $\int_{-1}^1 (x^{1/3} + 1) dx$
- (vii) Find the area above the x-axis and under the curve  $y = 5 - x^2$  from  $x = -1$  to  $x = 2$
- (viii) Solve differential equation  $ydx + xdy = 0$
- (ix) Find mid-point of line segment joining  $A(-8, 3)$ ;  $B(2, -1)$
- (x) Two points 'P' and 'O' given in xy-coordinate system. Find XY-coordinates of 'P' referred to translated axis  $O'X$  and  $O'Y$  for  $P(-2, 6)$ ;  $O'(-3, 2)$
- (xi) Find equation of the line joining  $(-5, -3)$  and  $(9, -1)$
- (xii) Find equation of vertical line through  $(-5, 3)$

(Turn Over)

## 4. Write short answers to any NINE (9) questions :

- (i) Graph the solution set of given linear inequality in  $xy$ -plane :  $2x + y \leq 6$
- (ii) Find the centre and radius of the circle with the given equation  
 $5x^2 + 5y^2 + 14x + 12y - 10 = 0$
- (iii) Find the focus and vertex of the parabola  $x^2 = -16y$
- (iv) Write an equation of parabola with given elements : Focus  $(-3, 1)$  ;  
 directrix  $x - 2y - 3 = 0$
- (v) Find an equation of directrices of given hyperbola  $\frac{x^2}{4} - \frac{y^2}{9} = 1$
- (vi) Find the centre and eccentricity of given hyperbola  $\frac{y^2}{16} - \frac{x^2}{9} = 1$
- (vii) Find the unit vector in the same direction as the vector  $\underline{v} = [3, -4]$
- (viii) Find the constant  $a$  so that the vectors  $\underline{v} = i - 3j + 4k$  and  $\underline{w} = ai + 9j - 12k$  are parallel.
- (ix) Find a vector of length 2 in the direction opposite that of  $\underline{v} = -i + j + k$
- (x) Find the cosine of the angle  $\theta$  between  $\underline{u}$  and  $\underline{v}$   $\underline{u} = [2, -3, 1]$  and  $\underline{v} = [2, 4, 1]$
- (xi) Compute  $\underline{b} \times \underline{a}$ . Check your answer by showing that  $\underline{b}$  is perpendicular to  $\underline{b} \times \underline{a}$  :  
 $\underline{a} = 2i + j - k$  ;  $\underline{b} = i - j + k$ .
- (xii) If  $\underline{a} + \underline{b} + \underline{c} = 0$ , then prove that  $\underline{a} \times \underline{b} = \underline{b} \times \underline{c} = \underline{c} \times \underline{a}$
- (xiii) Give a force  $\underline{F} = 2i + j - 3k$  acting at a point A  $(1, -2, 1)$ . Find the moment of  $\underline{F}$  about the point B  $(2, 0, -2)$

## SECTION - II

Note : Attempt any THREE questions.

5. (a) Find value of  $k$ , if the function  $f(x) = \begin{cases} \frac{\sqrt{2x+5} - \sqrt{x+7}}{x-2} & , x \neq 2 \\ k & , x = 2 \end{cases}$  is continuous at  $x = 2$  5
- (b) If  $y = \tan(p \tan^{-1} x)$  then show that  $(1+x^2)y_1 - p(1+y^2) = 0$  5
6. (a) Evaluate  $\int \frac{\sqrt{2}}{\sin x + \cos x} dx$  5
- (b) Find an equation of the line through the intersection of the lines  $x - y - 4 = 0$  and  $7x + y + 20 = 0$  and parallel to the line  $6x + y - 14 = 0$  5
7. (a) Find the area bounded by the curve  $y = x^3 - 4x$  and the  $x$ -axis. 5
- (b) Maximize  $f(x, y) = 2x + 5y$  subject to the constraints  $2y - x \leq 8$ ,  $x - y \leq 4$ ,  $x \geq 0$ ,  $y \geq 0$  5
8. (a) Write equation of the circle passing through the points A  $(-7, 7)$ , B  $(5, -1)$  and C  $(10, 0)$  5
- (b) Find a vector of length 5 in the direction opposite that of  $\underline{v} = i - 2j + 3k$  5
9. (a) Show that  $y = \frac{\ln x}{x}$  has maximum value at  $x = e$  5
- (b) Find focus, vertex and directrix of parabola  $x^2 - 4x - 8y + 4 = 0$  5